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EVALUATION OF IRON ISOTOPES SCATTERING CROSS SECTION USING PARTIAL WAVE EQUATION

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ABSTRACT

In this paper, the scattering of Iron was observed from a spherically symmetric potential as a typical example of the problems studied in computational physics. Scattering experiments are perhaps the most important tool for obtaining detailed information on the structure of matter, in particular the interaction between particles. The differential and total scattering cross section of the particles were calculated, by using Partial wave equation, written using FORTRAN. This research concentrated on $^{56}{}_{26}$Fe (atomic mass number $A = 56$) with energies within the range of $1 \leq E \leq 50\ \mathrm{MeV}$ and angular momentum ($1 \leq l \leq 18$). It was found that the value of the differential cross section and total cross section has the highest probability of interaction at 3800barn and 41293barn for $^{56}Fe_{26}$ isotopes and lowest scattering was observed at 0 barn respectively, for same angular momentum. For $l = 1$ the interaction does not depend on the scattering angle but for $l = 0$ there was no interaction.

Keyword: Atomic Number, Angular Momentum, Differential Cross Section, and FORTRAN.

1.0 INTRODUCTION

Scattering is a technique used in understanding nuclear forces and the laws that governs the interactions of elementary particles [1-3]. Differential cross section is the number of particles scattered into a given solid angle per unit time per unit incident flux. If a beam of particle is incident on a scattering object, it has been observed that some particles will be scattered while others pass undeviated and hence the probability that scattering took place is defined, sometimes our interest is not only in the total probability of scattering but also in the probability that the particle has scattered in a particular direction [4-13]. The differential cross section, describes how these intensities are distributed over the various spatial angles, and the integrated flux of the scattered particles is the total cross section(σ_{tot}). The neutron scattering cross section varies randomly between elements and even between different isotopes of the same element. One can thus use neutrons to study light isotopes. The interaction between neutrons and solids are weak, implying that neutrons in most cases probe the bulk of the sample, and not only its surface. In addition, quantitative comparisons between neutron scattering data and theoretical models are possible, since higher order effects are small and can usually be corrected for, or neglected. Since neutrons penetrate matter easily, neutron scattering can be performed with samples stored in all sorts of sample environment: Cryostats, magnets, furnaces, pressure cells, etc. Furthermore, very bulky samples can be studied, up to 10 cm thickness, depending on its elemental composition [14-20]. Also, differential scattering cross section is an important quantity in neutron activation and radioisotope production, these experimental quantities are what were calculated [21&22].

However, for this research, the obtained results were compared with theoretical estimates simulated by the partial wave analysis method, using computer simulation (KAP19) Fortran programme, which was used to obtain the result.

1.2 Theoretical Background

Equation (1) gives the formula used in calculating the energies of the various isotopes [3, 4 & 6].

$$E = \frac{2\hbar^2\pi^2 n^2}{ma^2} \tag{1}$$

The angular momentum of Iron was also calculated using equation (2)

$$l = (n-1) \tag{2}$$

with Iron having a maximum angular momentum of three.

The total cross section for individual isotopes was calculated using equation (4)

$$\sigma = 2\pi \sum_i \frac{\partial\sigma}{\partial\Omega_i} \theta_i \sin\theta_i d\theta_i \,. \tag{4}$$

2.0 Materials and Methods

This research made use of computer programme (KAP19) FORTRAN, where the input data was compiled, built, and executed. The subroutines for this programme are BESJ for computing spherical Bessel function of second kind (BESN) and Legendre polynomial (PLM) with the function subroutine FAK. The notation that was employed in the function subroutine V,as shown in Table 1.

Table 1: Notation of the subroutine

Physical title	Fortran title	Physical title	Fortran title
V_1	V1	E	E
V_2	V2	A	A
V_3	V3	Z	Z
a	A0	r	R
r_0	R0	R	R1
V_0	V0		

Here *A* is the mass number and *Z* the charge number of the target nucleus, while *E* is the kinetic energy of the incident neutron. The parameter values are valid in the region $1 \leq E \leq 50 MeV$and $A \geq 40$. The neutron input data are the mass number *A*, the nuclear charge number *Z* of the target nucleus, the energy E of the neutron and the angular momentum quantum number, at which the summation over angular momentum partial waves is truncated. As output, one obtains the differential cross-section ($\frac{d\sigma}{d\Omega}$) and the scattering angle(θ). The output of $\frac{d\sigma}{d\Omega}$ as a function of the scattering angle is obtained in graphical form. Two diagrams was shown in the graphical output: in the first graph is the differential cross section($\frac{d\sigma}{d\Omega}$), which was plotted against the scattering angle, while the second graph log of the differential cross section was plotted against the scattering angle [22-27].

2.1 Materials

In the process of solving the scattering cross section of iron isotopes some of the following will be employed in accordance with [27-34]: Modelled equation, Co mputer system. FORTRAN Programme (KAP19), Excel package, GNU plot .

2.2 Method

The Fortran software was installed in the computer system, the programme KAP19 Fortran was already in the system, some comments were removed from the programme, a file directory were added to different lines 180, and the programme was built and executed for errors. The following modifications were made to the programme.

1. Variables of string data type were created at line 67. These variables (parameters) are AA, ZZ, EE and LLP, which combine together to form file name at line 188 to 189 in order to give a different file name any time it's been run. Also, not to replace the previous data anytime a new value is run.
2. Variable A is stored in AA as string, Z is stored in ZZ as string, E is stored in EE as string and LP is stored in LLP as string.
3. At line 180, a log file was added in order to convert the values in A, Z, E, LP into string which were used by AA, ZZ, EE and LLP
4. At line 180 the directory was created "Users\Usman Oyiza\Desktop\F PROJ\F\with a name log.dat".
5. A code was added which write A, Z, E and LP to the log file.
6. At line 182, a code was written to close the file. "close (10)"
7. From line 184 to 186, a code was added so as to pass values to parameter AA, ZZ, EE and LLP.
8. File with dynamic name were coded at line188 and 189.
9. Values were written to the dynamic files at line 200 and 201and the file was closed at line 215.
10. The codes at line 216-219 were modified by changing what is printed on the screen from "Rerun program for A =…." to "Do you want to rerun the program…." same changes were made in line 227-229 and 237-239
11. From line 241-246, a programme was added to change the values in the log file to new values any time the program is rerun to a new value without exiting the program to start again.
12. At line 206, a function call system('Cls') was added to clear the screen.

In running the programme, the mass number of the incident particles, its charge number, angular momentum and calculated energies were inputted simultaneously. The programme calculated various differential cross section for the isotope of the element, ${}^{56}_{26}$Fe. The values of the data were stored in the file created, after the whole data was run, the files which contains the calculated scattering angles and differential cross section where renamed and copied to excel sheet. It was later compiled and then run. After calculating the total cross section using Excel, the graph was plotted as total cross section against the scattering angle and the differential cross section against scattering angle respectively using a GNU Plot package which gave the result of the research.

2.3 Data for Iron Isotopes

The parameters used for iron isotopes are given in Table 2

Table 2: Data for iron isotopes

S/N	Iron isotopes	Angular momentum (l)	Energies (E) MeV
1	$^{54}Fe_{26}$	0	15.2929
2		1	3.82322
3		2	1.6990
4		3	0.9558
5	$^{56}Fe_{26}$	0	14.747
6		1	3.7007
7		2	1.63852
8		3	0.92167
9	$^{57}Fe_{26}$	0	14.488
10		1	3.6220
11		2	1.6078
12		3	0.9055
13	$^{58}Fe_{26}$	0	14.238
14		1	3.5596
15		2	1.5820
16		3	0.8899

3.0 Results and Discussion

The results obtained are represented in graphs for element and their various isotopes of Iron $\left({}^{56}_{26}Fe\right)$. It shows the result for same values of energies but different angular momentum and result for same angular momentum but different energies respectively.

3.1 Same Energies but Different Angular Momentum

Figure 1 gave the differential crossectional result discussion in relation to the angular scattering. The maximum value of the differential crossection to be 3600 mb with angular scattering ranging from 0-180^0.

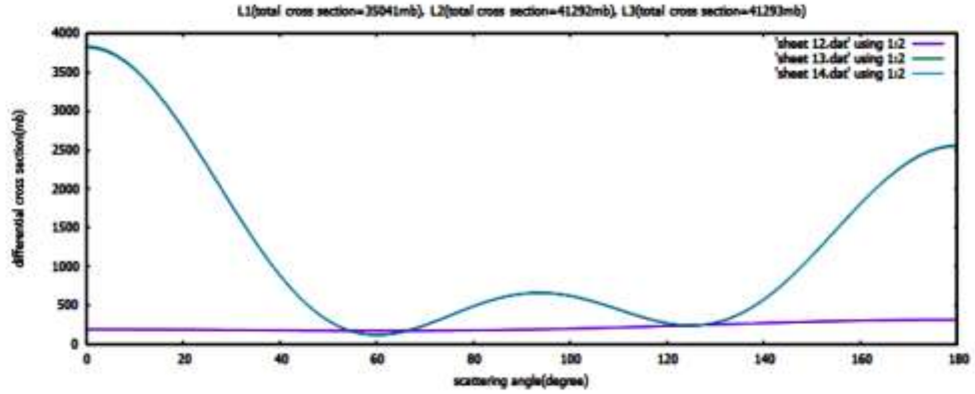

Figure 1: Graph of differential and total cross section against scattering angle with energy $E_1=0.9217 MeV$ and angular momentum (l =1, 2 and 3) for ${}^{56}_{26}Fe$ isotope.

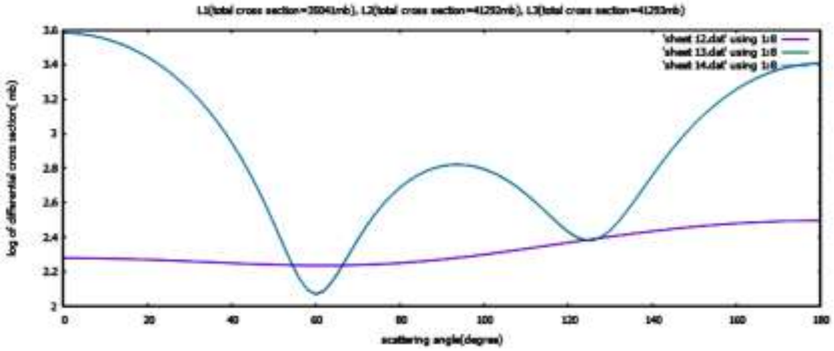

Figure 2: Graph of log of differential cross section against scattering angle with energy E$_1$=0.9217MeV and angular momentum (l =1, 2 and 3) for $^{56}_{26}Fe$ isotope.

Figure 1 and 2 gave the result for the total scattering cross section and differential cross section with energy E=0.9217MeV and angular momentum (l =1, 2 and 3) for $^{56}_{26}Fe$ $i$sotope. It could be seen that the total scattering cross section and differential cross section have the highest probability of interaction at 41293barn and 4000barn respectively, which was given by the angular momentum l =2 and 3 and a lowest scattering was observed at a scattering angle of angle of 60˚,the higher the angular momentum the higher the total cross section.

3.2 Same Angular Momentum but Different Energies

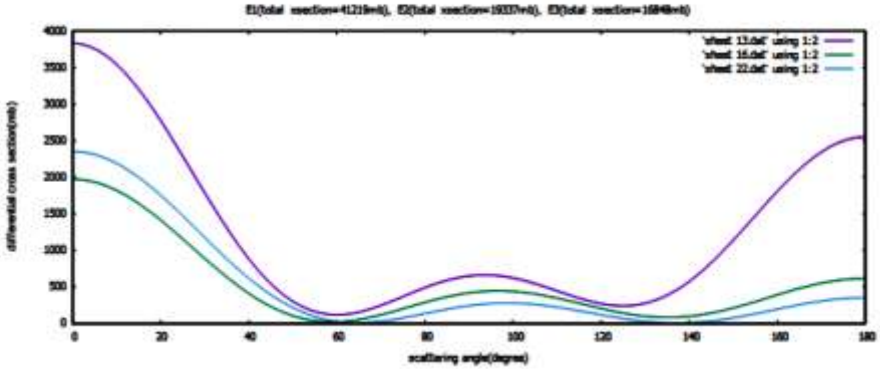

Figure 3: Graph of differential and total cross section against scattering angle with the angular momentum $l = 2$ and Energies: E$_3$ = 1.6385MeV, E$_2$= 3.7001MeV, E$_1$ = 14.747MeV for $^{56}_{26}Fe$ isotope

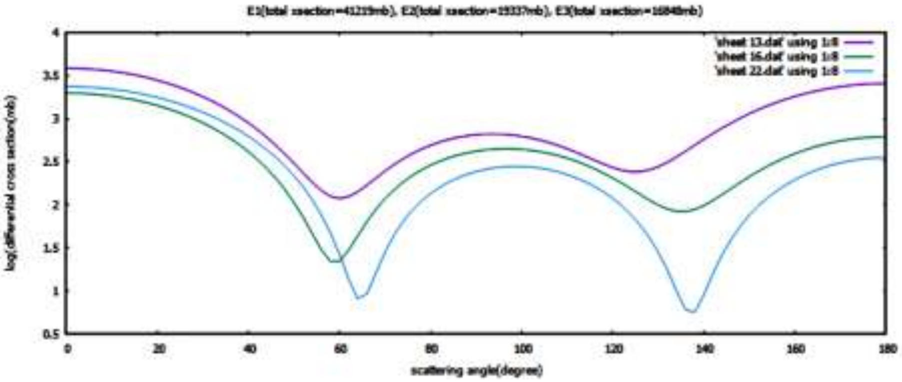

Figure 4: Graph of log of differential cross section against scattering angle with the angular momentum $l = 2$ and Energies: E$_3$ = 1.6385MeV, E$_2$= 3.7001MeV, E$_1$ = 14.747MeV for $^{56}_{26}Fe$ isotope.

Figure 3 and 4 gave the results for the total cross section and differential cross section with the angular momentum $l = 2$ and Energies: E_3 = 1.6385MeV, E_2= 3.7001MeV , E_1 = 14.747MeV for $^{56}_{26}Fe$ isotope. It could be seen that the variation of energies affects the cross section, the lower the internal energy leads to decrease in total cross section.

3.3: Results for $^{56}_{26}Fe$ Isotope at Angular Momentum (l = 2 , 4 and 5) at 10MeV

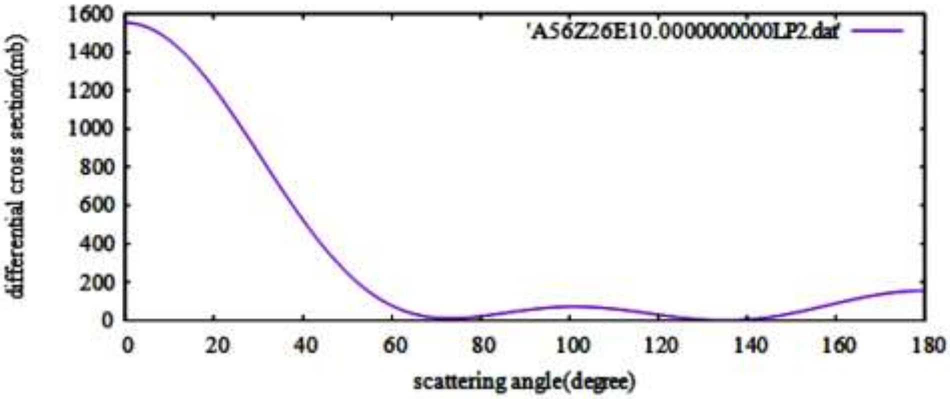

Figure 5: Differential cross-section for the scattering of 10MeV neutrons for $^{56}_{26}Fe$ (l = 2), (this work).

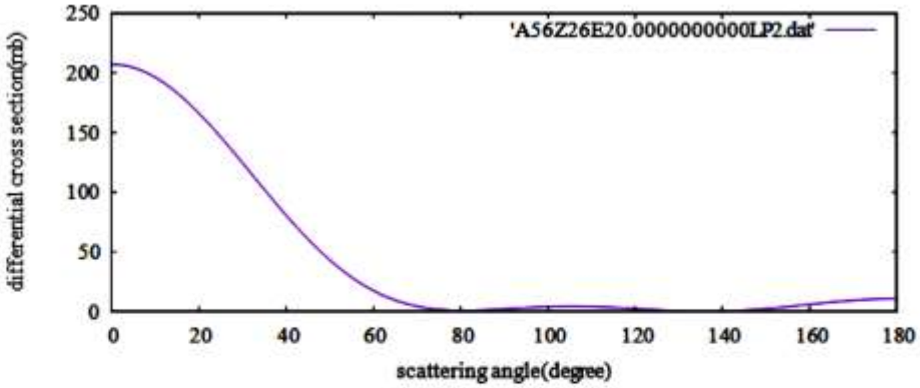

Figure 6: Differential cross-section for the scattering of 10MeV neutrons for $^{56}_{26}Fe$ (l = 4), (this work).

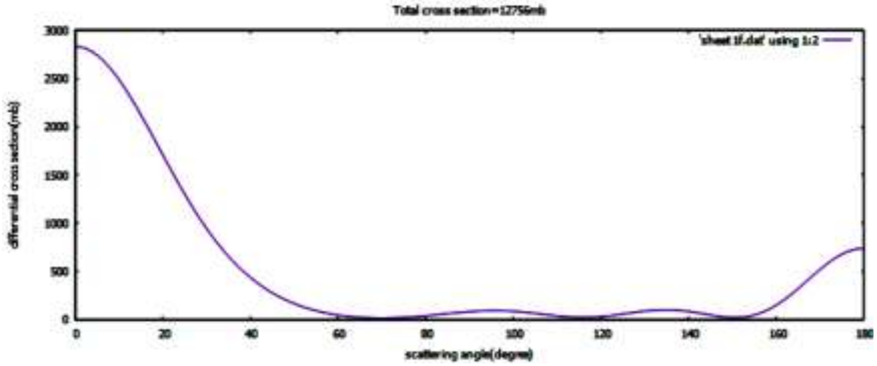

Figure 7: Differential and total cross-section for the scattering of 10MeV neutrons from $^{56}_{26}Fe$(l_p = 5), this work

4.0 Discussion

The investigation of the effect of angular momentum on the scattering cross section of Iron from a spherically symmetric potential reveals that, with increasing values of l, the total cross section and differential cross-sections increases. The targeted nucleus is Iron with energies less than 50 MeV and angular momentum $l < 18$. It could be seen that from figure 1 to 2 for same energies but different angular momentum, the variation of angular momentum affects the total cross section of Iron,the higher the angular momentum the higher the total cross section of the Iron isotopes and also, from figure 3 to figure 4 for same angular momentum but different energies, the variation of energies affects the total cross section of Iron isotopes, the lower the internal energy the lower the total cross section. In figure 5 to figure 7 the differential cross section for the scattering of neutrons by $^{56}_{26}Fe$ at an incident energy of $10MeV$, at angular momentum of 2, 4 and 5 respectively. It could be seen that the differential cross section has the highest probability of interaction at 1600 barn, 200 barn and 2800 barn respectively and a lowest scattering was observed at a scattering angle of 60˚, 70˚, 120˚and 150˚.

5.0 Conclusion

The calculation of the scattering cross section of some selected particles from a spherically symmetric potential at E < 50MeV and A > 40 were investigated using KAP 19 Fortran code. The total scattering cross-section for Iron target was calculated based on the results of the differential cross section gotten using KAP 19 Code. It was found that the scattering cross section has the highest probability of interaction at □ = 3 and □ = 5 respectively for Iron and independent on the scattering angle at $l = 0$. So, interaction does not occur on the different target nucleus. Therefore, investigating the scattering of some selected particles from a spherically symmetric potential, the partial wave analysis method was found to be effective and the KAP 19 Fortran code used in calculating the differential cross section gave accurate result.

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